Hale School

Year 11 Semester 1 Examination, 2015

Write your name below:

MARKING KEY

Mathematics Methods

Teacher:_____

Section One: Calculator-free

Booklet 1 of 3

TIME ALLOWED FOR THIS PAPER

Reading time before commencing: Five minutes
Working time for paper: Fifty minutes

MATERIAL REQUIRED/RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR
This Question/Answer booklet.
Formula Sheet.

For Examiners only	
Section 1	
Section 2	
Total	

TO BE PROVIDED BY THE CANDIDATE

Standard Items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special Items: nil

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. Please check carefully, and if you have any unauthorised material with you, hand it to the supervisor **BEFORE** reading any further.

STRUCTURE OF THIS PAPER

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	9	9	50	50	37
Section Two: Calculator-assumed	15	15	100	85	63
				Total	100

INSTRUCTIONS TO CANDIDATES

1. Write your answers in this Question/Answer Booklet.

2. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

3. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

4. Fill in the number of the question that you are continuing to answer at the top of the page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

Section One: Calculator Free 50 marks (37%)
This section has 8 questions. Answer all questions. Write your answers in the spaces provided.
Working time: 50 minutes

Question 1 **5 marks**

a. State the exact value of $\sin(\frac{2\pi}{3})$.

$\frac{\sqrt{3}}{2}$

✓ States solution

b. State the exact value of $\cos(225°)$.

$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

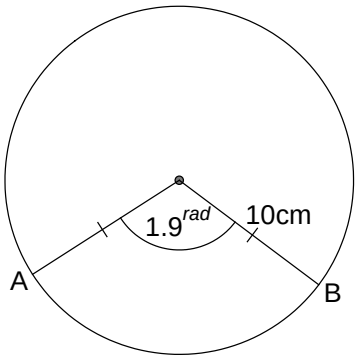
✓ States solution

c. Convert $18°$ to radians.

$\frac{\pi}{10}$

✓ States solution

d. Determine the length of the minor arc AB shown below.



19 cm

✓ States solution

e. Factorise $x^2 - 7x + 10$

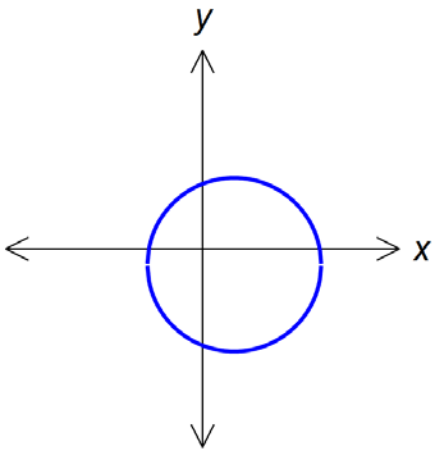
$(x-5)(x-2)$

✓ States solution

Question 2 **3 marks**

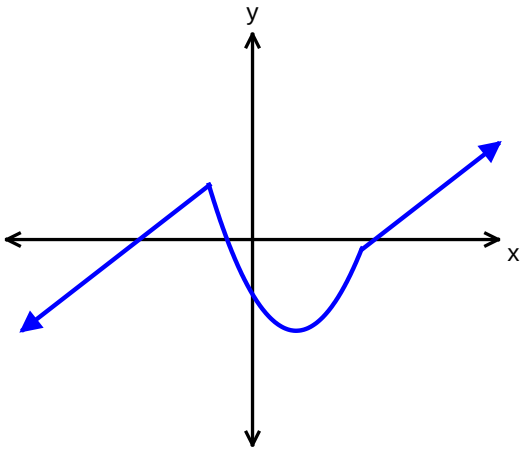
State whether each of the following is a graph of a function or not.

a.



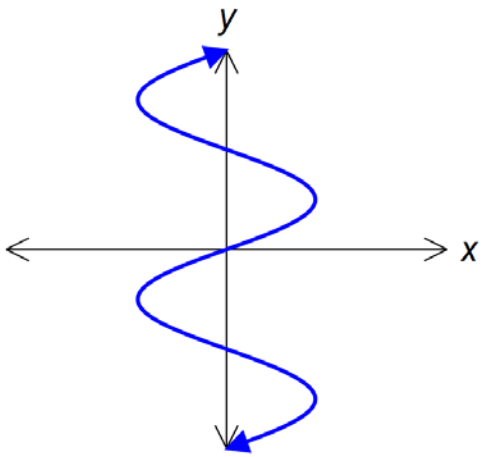
✓ Not a Function

b.



✓ Function

c.



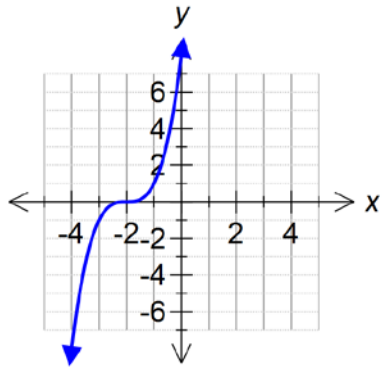
✓ Not a Function

Question 3 **6 marks**

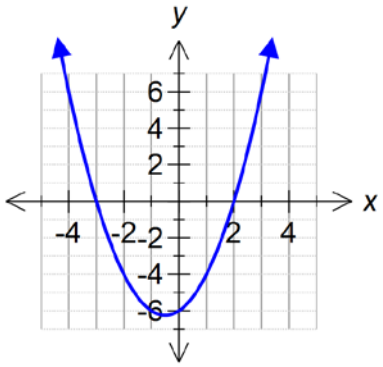
Some of the equations listed below match up to each of the graphs shown. Write the correct equation next to the relevant graph.

A $y = x^5 - 10x^4 - 10x^3 + 200x^2 + 20x$		B $y = 10x^4 - 10x^3 + 200x^2 + 20x$	
C $y = (x-2)(x+3)$	D $y = \frac{1}{x-3} + 2$	E $y = (x+2)^3$	F $y = -\sqrt{x-2}$
G $y = (x+2)(x-3)$	H $y = \frac{1}{x+3} + 2$	I $y = 3x(x+2)^2$	J $y = \sqrt{-x+2}$

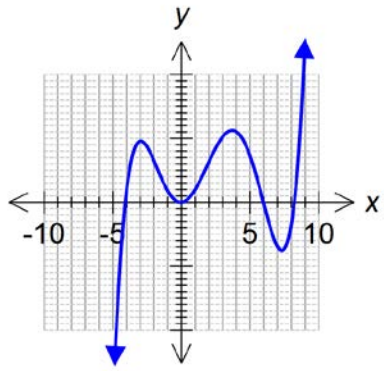
a. ✓E $y = (x+2)^3$



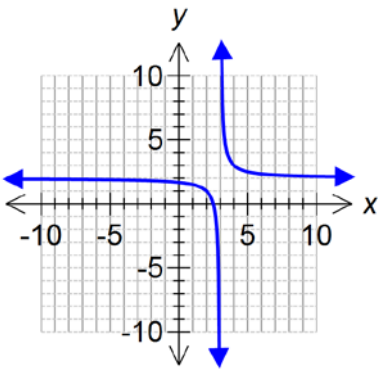
b. ✓C $y = (x-2)(x+3)$



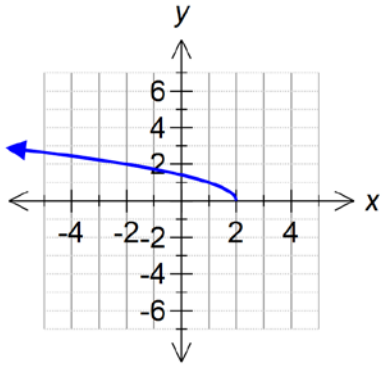
c. ✓A $y = x^5$ ...



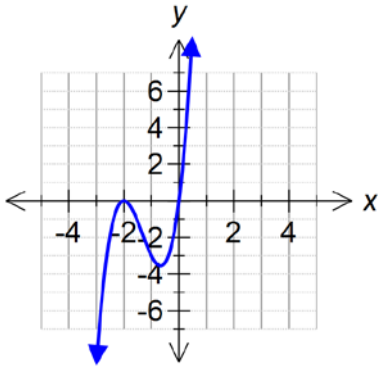
d. ✓D $y = \frac{1}{x-3} + 2$



e. ✓J $y = \sqrt{-x+2}$



f. ✓I $y = 3x(x+2)^2$



Question 4 **6 marks**

Solve each of the following equations for x.

a. $\frac{11x+7}{5} = 7 + 3x$ (2 marks)

$11x + 7 = 35 + 15x$

$-4x = 28$

$x = -7$

✓ Gathers like terms correctly.

✓ Applies inverse operations correctly to solve for x.

b. $3x^2 = 12x$ (1 mark)

$3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0$ or $x = 4$

✓ States both solutions

c. $x^2 + 4x - 3 = 0$ (3 marks)

$(x+2)^2 - 4 - 3 = 0$

$(x+2)^2 = 7$

$x + 2 = \pm\sqrt{7}$

$x = -2 \pm \sqrt{7}$

✓ Chooses appropriate method to solve (complete square/quad formula).

✓ Correctly applies chosen method.

✓ States correct solutions.

Question 5 **6 marks**

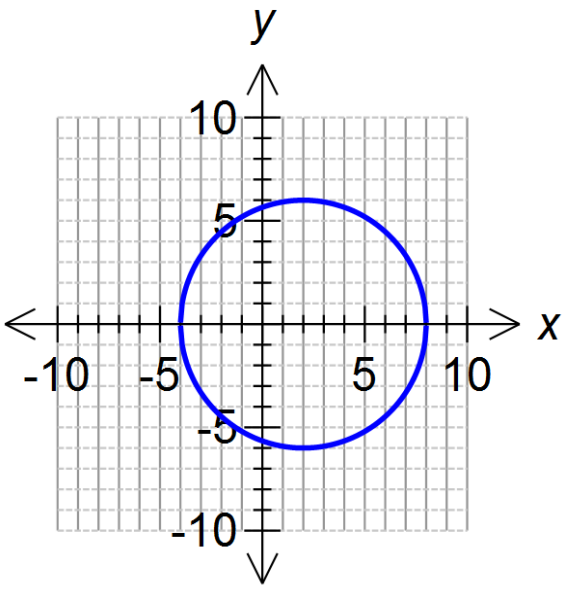
State the equation of each graph shown below.

a. $(x-2)^2+y^2=6^2$ (3 marks)

✓ Uses correct notation and form of equation

✓identifies relationship between centre point and equation

✓Identifies relationship between radius and equation

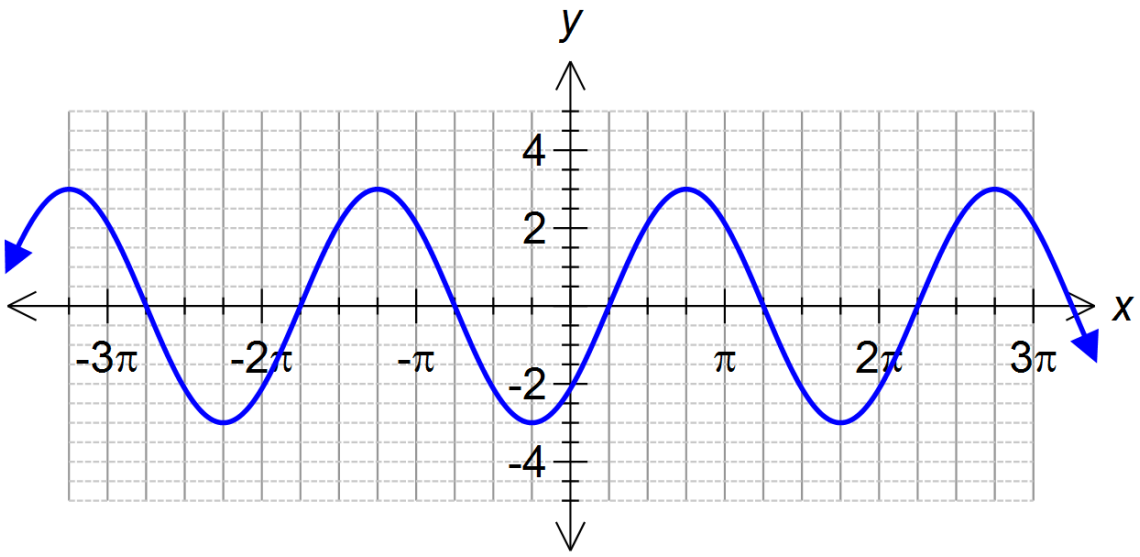


b. $y=3\sin(x-\frac{\pi}{4})$ or $y=3\cos(x-\frac{3\pi}{4})$ (3 marks)

✓ Uses correct notation and form of equation

✓identifies relationship between amplitude and equation

✓Identifies relationship between phase shift and equation



Question 6 **4 marks**

The first four rows of Pascal's triangle are shown below.

1
1 1
1 2 1
1 3 3 1

a. Expand the expression $(x+3)^4$, leaving terms in index form where necessary. (2 marks)

$1x^4 + 4(3)x^3 + 6(3^2)x^2 + 4(3^3)x + 1(3^4)$

✓Determines correct coefficients

✓Uses coefficients to write expanded expression

b. Fraser won a competition to take himself and 3 friends to the Grand Final. He has 6 close friends who want to go with him. How many different combinations of 3 friends are there for Fraser to choose from? (2 marks)

✓Uses appropriate method (formula or triangle or list?)

✓States solution nCr = 20

$n=6 \rightarrow$ 1 6 15 20 15 6 1

${}^6C_0$ ${}^6C_1$ ${}^6C_2$ ${}^6C_3$

${}^6C_3 = 20$

Question 7 **6 marks**

a. Solve $4\cos^2 x - 3 = 0$, $for\ 0° \leq x \leq 360°$ (3 marks)

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

✓ Correctly rearranges to make cos x the subject.

✓ Determines correct reference angle.

✓ States all solutions for the domain and reference angle.

$x = 30°, 150°, 210°, 330°$

b. Completely factorise the function $f(x)$ below, given that $f(3) = 0$. (3 marks)

$$f(x) = x^3 - 6x^2 - x + 30$$

$f(x) = (x-3)(x^2-3x-10)$

$= (x-3)(x-5)(x+2)$

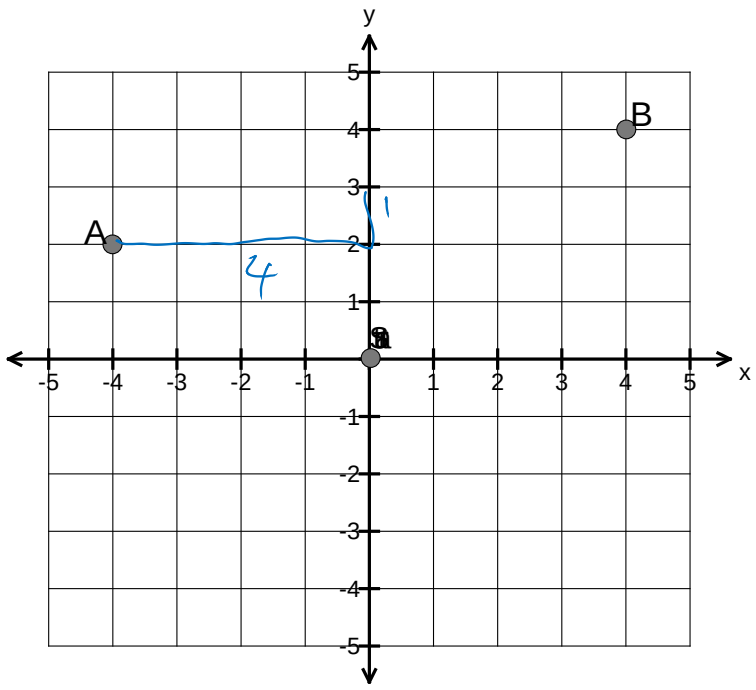
✓ Recognises (x-3) is a factor

✓ Removes linear factor correctly

✓ Factorises remaining quadratic correctly

Question 8 **8 marks**

Jacob is on a treasure hunt but only has part of the map to guide him. He must walk from his starting point at the origin to each of the markers A, B and C, before reaching the treasure at X. The map below shows his starting point, the first marker at A and the second marker at B.



a. Jacob reaches point A and must travel to point B in a straight line. Determine the equation of the line passing through points A and B. (3 marks)

$y = \frac{1}{4}x + c$

$B(4,4)$ $4 = \frac{1}{4}(4) + c$

$c = 3$

$y = \frac{1}{4}x + 3$

✓*Determines gradient*

✓*Determines y-intercept*

✓*States equation based on values calculated*

Jacob knows that points A, B and C lie in a straight line. When he reaches point B, he has travelled halfway along the line segment AC.

b. Determine the coordinates of point C. (2 marks)

$\left(\frac{-4+C_x}{2}, \frac{2+C_x}{2}\right) = (4, 4)$

✓ Recognise midpoint/endpoint relationship

$C = (C_x, C_y) = (12, 6)$

✓ Determine coordinates of C

c. To reach point X, Jacob must then travel along a line that is perpendicular to AB. Determine the gradient of this line. (1 mark)

$m = -4$

✓ States correct gradient
(allow follow through from part a)

d. Harry is also on the treasure hunt and knows that the coordinates of the treasure at X are $(\frac{17}{2}, \frac{17\sqrt{3}}{2})$ and decides to go straight there from the starting point. Determine the angle that the line Harry travels on will make with the x-axis on the map. (2 marks)

✓ Recognise relationship of gradient to angle

✓ Calculate correct angle

$\tan\theta = \frac{y}{x}$

$= \frac{17\sqrt{3}}{2} \div \frac{17}{2}$

$\tan\theta = \sqrt{3}$

$\theta = 60°$

Question 9 **6 marks**

The function $f(x) = x^2 - 3kx + 2k^2$ is defined for all $k \in \mathbb{R}$.

a. If $k = 2$, determine the coordinates of the y-intercept of $f(x)$. (1 mark)

$(0, 8)$

✓ States the y-intercept
(one mark off whole question only for not writing coordinates)

b. If $k = 3$, determine the coordinates of the x-intercepts of $f(x)$. (2 marks)

$f(x) = x^2 - 9x + 18$

$= (x-6)(x-3)$

✓ Substitutes k=3 correctly

✓ Factorises expression

✓ States intercepts

$\therefore x = 6$ or $x = 3$

$(6, 0)$ and $(3, 0)$

c. For what values of k will $f(x) = 0$ have 2 solutions? Justify your answer. (3 marks)

2 solutions $\Rightarrow b^2 - 4ac > 0$

$(-3k)^2 - 4(1)(2k^2) > 0$

$9k^2 - 8k^2 > 0$

$k^2 > 0$

$\therefore k > 0$ or $k < 0$

i.e. $\{k \in \mathbb{R} : k \neq 0\}$

✓ Identifies $\Delta > 0$

✓ Substitutes a, b and c in terms of k correctly

✓ Determines range of possible values for k

END OF SECTION ONE